

## MODULE 2.

### PRINCIPLES OF 3D LASER SCANNING

#### Learning Outcomes:

This module discusses the principles of electronic distance measurement (EDM) and 3D laser scanning. Student should be able to explain how distance is measured in EDM and 3D laser scanning as well as the impact of index of refraction and to perform related calculations.

#### Lecture Contents:

##### 2.1 *Electromagnetic Radiation*

All electronic distance measurement (EDM) devices, laser included, use electromagnetic radiation. In modern physics, it was observed that electromagnetic radiation behaves in two complementary ways: as electromagnetic waves and as a stream of massless particles called photons. In EDM, electromagnetic radiation is generally described with electromagnetic waves. Electronic waves have wave length ( $\lambda$ ), and frequency ( $f$ ) and carry energy ( $E$ ). Figure 2.1 shows a form of waves and wavelength.

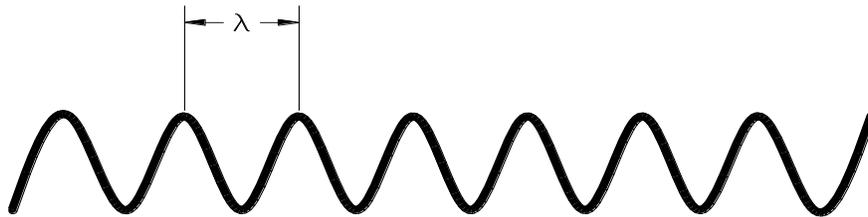


Figure 2.1 Illustration of electromagnetic waves and wavelength.

Frequency is the number of waves passing a fixed point in a given time. The propagating velocity of electromagnetic waves ( $V$ ), commonly known as the speed of light, is 299,792,458 m/sec in a vacuum. The relationship among  $V$ ,  $\lambda$ , and  $f$  is as follows:

$$\lambda = \frac{V}{f} \quad (2.1)$$

The energy that an electromagnetic wave carries can be calculated as:

$$E=hf \quad (2.2)$$

where  $h$  is the Plank's constant and is equal to  $4.135 \times 10^{-15}$  eV-sec (electron volts-sec) or  $6.626069 \times 10^{-34}$  J-sec.

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### Example 2.1

Green laser used in some 3D laser scanners has a wavelength of 532 nm, calculate the energy that the wave carries (one photon) in a vacuum.

Solution:

From Equations 2.1 and 2.2:

$$E = hf = h \frac{V}{\lambda} = 6.626069 \times 10^{-34} \times \frac{299792458}{5.32 \times 10^{-8}} = 3.734 \times 10^{-18} \text{ J.}$$

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The range of all electromagnetic radiation with different wavelengths, hence different frequencies and energy levels, is known as the electromagnetic spectrum, or the spectrum. Figure 2.2 shows the spectrum with names of radiations, wavelengths, frequencies, energy levels, and emission sources. It can be seen from this figure that Tellurometers are generally in the microwaves range while laser distance measurement instruments are in the ranges of near-infrared and visible lights.

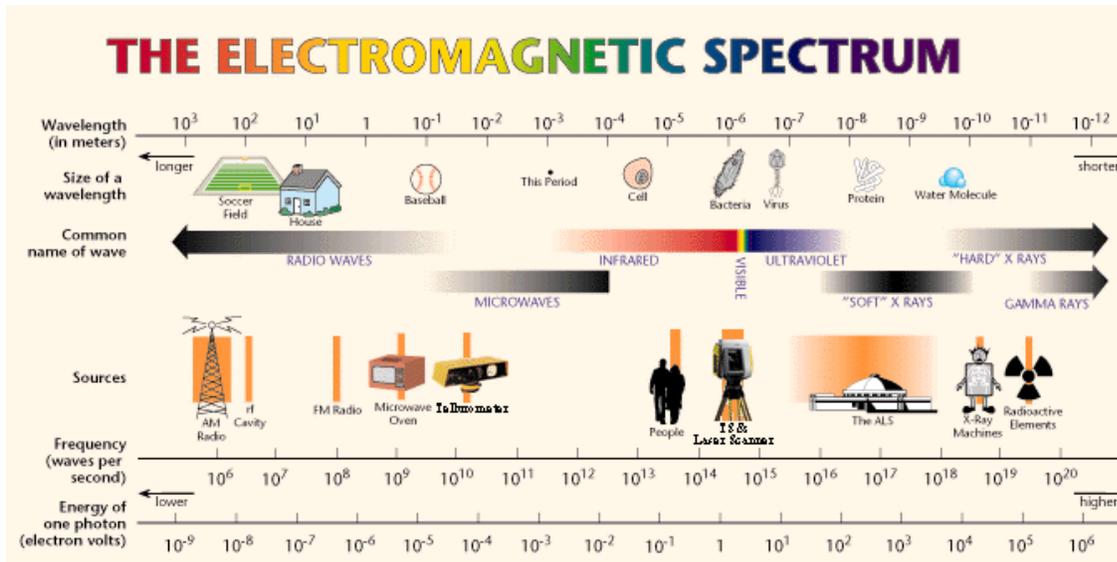


Figure 2.2 Electromagnetic spectrum with illustrations showing the EM radiation properties of different objects. Note the frequency ranges for the Tellurometer, TS and 3D scanners. (Modified from the Lawrence Berkeley National Laboratory website)

## 2.2 Electromagnetic Propagation Rate and the Index of Refraction

The accuracy of electronic distance measurement depends on the accuracy of the electromagnetic propagating rate or wavelength. In the atmosphere, the electromagnetic propagating rate is affected by temperature, atmospheric pressure, humidity and other factors, and therefore, lower than that in a vacuum. The ratio between the propagation rates in a vacuum and in atmosphere is the index of refraction which can be expressed as follows:

$$n = \frac{c}{V} \quad (2.3)$$

where  $c$  is the speed of light in a vacuum and  $n$  is the index of refraction. The value of  $n$  is around 1.0003. The accuracy of this constant is very important in precise surveying as shown later in the example. Various empirical formulas have been used to calculate it. The Edlen formula and its modified versions are among the most popular ones. Swedish physicist Bengt Edlen developed the original formula in 1953 and the formula has gone through a series of modifications over the years by various scientists. The latest version by Birch, Downs, Stone and Zimmerman (2005) is shown as follows:

$$S = \frac{1}{\lambda^2}; \quad (2.4)$$

$$n_s = 1 + 10^{-8} \left( 8342.54 + \frac{2406147}{130 - S} + \frac{15998}{38.9 - S} \right); \quad (2.5)$$

$$X = \frac{1 + 10^{-8} (0.601 - 0.00972t)P}{1 + 0.00366t}; \text{ and} \quad (2.6)$$

$$n = 1 + \frac{P(n_s - 1)X}{96095.43} - 2.9275 \times 10^{-8} \frac{(3.7345 - 0.04015)P_v}{t + 273.15} \quad (2.7)$$

where  $\lambda$ ,  $P$ ,  $t$  and  $n$  are the wavelength( $\mu\text{m}$ ), atmospheric pressure (Pascal), temperature ( $^{\circ}\text{C}$ ) and index of refraction, respectively, and  $S$ ,  $n_s$  and  $X$  are intermediate variables.  $P_v$ , is the water vapor partial pressure (Pascal), and is estimated with the formula in this book below:

$$P_v = 10^{\left[ \frac{7.5t}{2373+t} + 0.78582 \right]} H \quad (2.8)$$

where  $h$  is humidity (%). Different formulas were used by Stone and Zimmerman to calculate  $P_v$  in their computer routine.

In a report to the International Association of Geodesy (IAG), Reuger recommended that for electronic distance measurement accuracy better than 1 ppm in the near-infrared and visible range, the Ciddor procedure be used and for accuracy less than 1 ppm, closed-form formulas (CFF) be applied. The Ciddor procedure included carbon dioxide concentration and mole fraction of water vapor in the calculation in addition to temperature, humidity, atmospheric pressure and wavelength. It was adopted as the basis of a new standard by the International Association of Geodesy (IAG). The procedure is quite tedious for manual calculations and computer routines are available for calculations. The formulas and constants involved in the Ciddor procedure are attached in Appendix I.

The closed-form formulas recommended by Reuger are:

$$N_g = 287.6155 + \frac{4.88660}{\lambda^2} + \frac{0.06800}{\lambda^4} \text{ and} \quad (2.9)$$

$$n = 1 + 10^{-6} \left( \frac{273.15 N_g P}{1013.25(t + 273.15)} - \frac{11.27 P_v}{t + 273.15} \right) \quad (2.10)$$

where  $N_g$  is the index of refraction for standard air which is defined as 0.0375%  $\text{CO}_2$  content at  $0^{\circ}\text{C}$  temperature, 1013.25 hPa pressure and 0 percent humidity. Unlike in Eqs. 2.6 and 2.7, the unit for pressure in Eqs 2.9 and 2.10 is hPa (100 pascals). Eqs. 2.9 and 2.10 were also used in Wolf and Ghilani's text book, *Elementary Surveying*.

Figures 2.3 through 2.5 and Tables 2.1 through 2.3 show the changes of refraction index as a function of temperature, humidity or atmospheric pressure as calculated by the modified Edlen, Ciddor and closed-form formulas. These figures and tables indicate:

1. The effect of temperature on the index of refraction is the most profound. For example, for a temperature change of 5 C° from 10 C° to 15 C° , the corresponding change in the index is more than 5 ppm when  $\lambda=0.915 \mu\text{m}$ ,  $P=104124 \text{ Pa}$ ,  $h=56$  and  $\text{CO}_2=380 \text{ ppm}$ .
2. The humidity has the least effect on the index of refraction. For a 10 percent increase in humidity from 30 to 40 percent, the change in the index is about 0.087 ppm when  $\lambda=0.915 \mu\text{m}$ ,  $P=104124 \text{ Pa}$ ,  $T=20 \text{ C}^\circ$  and  $\text{CO}_2=380 \text{ ppm}$ .
3. The modified Edlen formulas and the Ciddor method yield very similar results. In fact, they begin to differ only after seven decimal places and the difference is between 0.01 ppm and 0.02 ppm for the most parts. For most surveying applications, it makes no difference which method is used. Based on the 1985 FGCS (Federal Geodetic Control Subcommittee) standard, the most accurate surveying category is Order AA with an accuracy of 0.01ppm.
4. The closed-form formulas (CFF) differ significantly from the other two methods. The difference between the CFF and Ciddor is generally greater than 1 ppm. The Ciddor method is generally regarded as a more reliable method in estimating the index of refraction, but the calculations are more tedious. Since the modified Edlen formulas are not much more complicated than the CFF and the results are very close to those of the Ciddor method, they should be used if the Ciddor computing routine is not available.

From Eq. 2.1, the wavelength of an electromagnetic wave decrease as the propagating rate decrease since the wave energy does not change and the frequency is fixed.

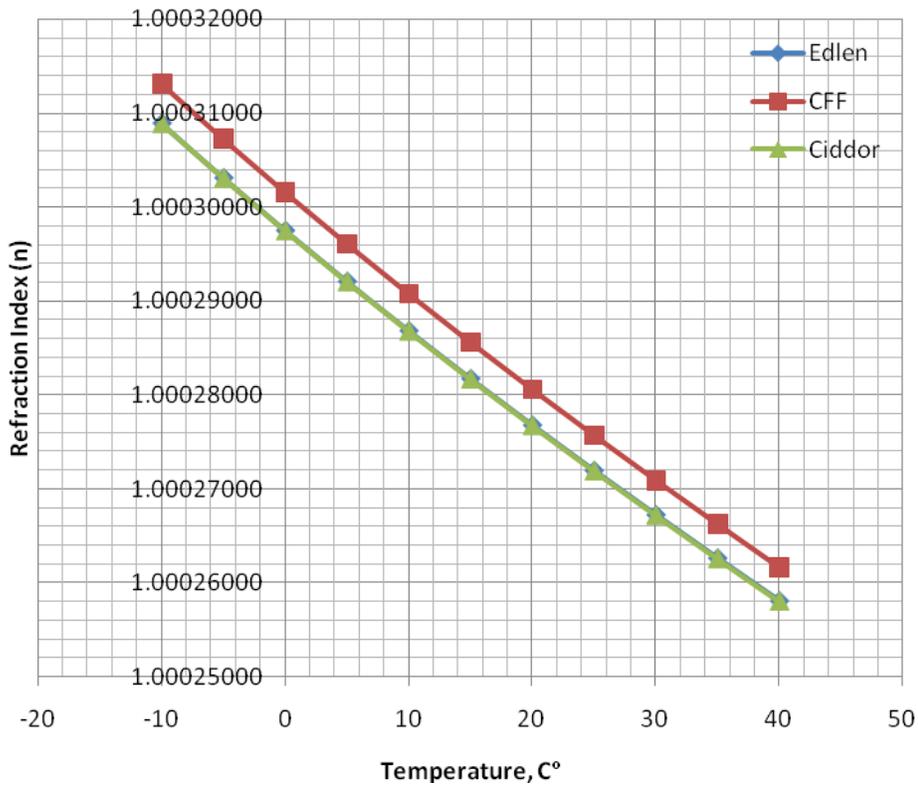


Figure 2. 3 Change of refraction index as a function of temperature

Table 2.1 Data used in Figure 2.3

( $\lambda=0.915 \mu\text{m}$ ,  $P=104124 \text{ Pa}$ ,  $h=56$  and  $\text{CO}_2=380 \text{ ppm}$ )

T (C°)	Edlen	CFF	Ciddor
-10	1.00030887	1.000313056	1.000308883
-5	1.00030306	1.000307187	1.000303069
0	1.00029746	1.00030152	1.000297454
5	1.00029204	1.000296041	1.000292031
10	1.00028679	1.000290734	1.00028678
15	1.0002817	1.000285584	1.000281687
20	1.00027675	1.000280577	1.000276737
25	1.00027193	1.000275697	1.000271914
30	1.00026722	1.000270926	1.000267202
35	1.0002626	1.000266247	1.000262584
40	1.00025805	1.000261643	1.000258042

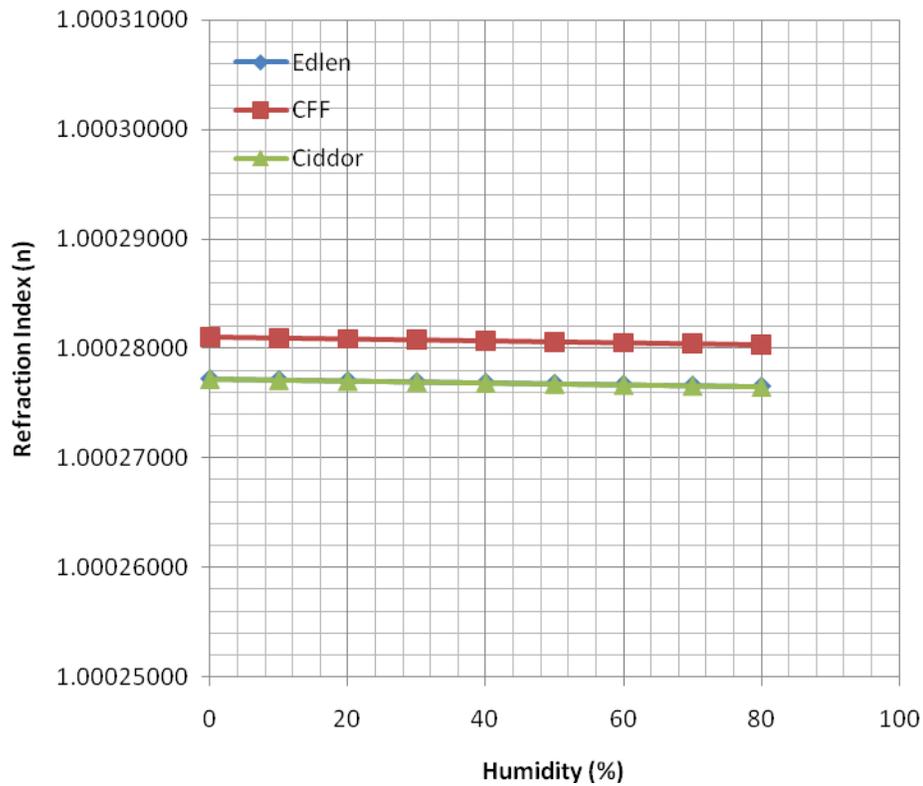


Figure 2. 4 Change of refraction index as a function of humidity.

Table 2.2 Data used in Figure 2.4

( $\lambda=0.915 \mu\text{m}$ ,  $P=104124 \text{ Pa}$ ,  $T=20 \text{ C}^\circ$  and  $\text{CO}_2=380 \text{ ppm}$ )

Humidity	Edlen	CFF	Ciddor
0	1.000277234	1.00028108	1.000277222
10	1.000277148	1.000280991	1.000277135
20	1.000277062	1.000280901	1.000277049
30	1.000276976	1.000280811	1.000276962
40	1.00027689	1.000280721	1.000276875
50	1.000276804	1.000280631	1.000276789
60	1.000276718	1.000280541	1.000276703
70	1.000276632	1.000280452	1.000276617
80	1.000276546	1.000280362	1.000276531

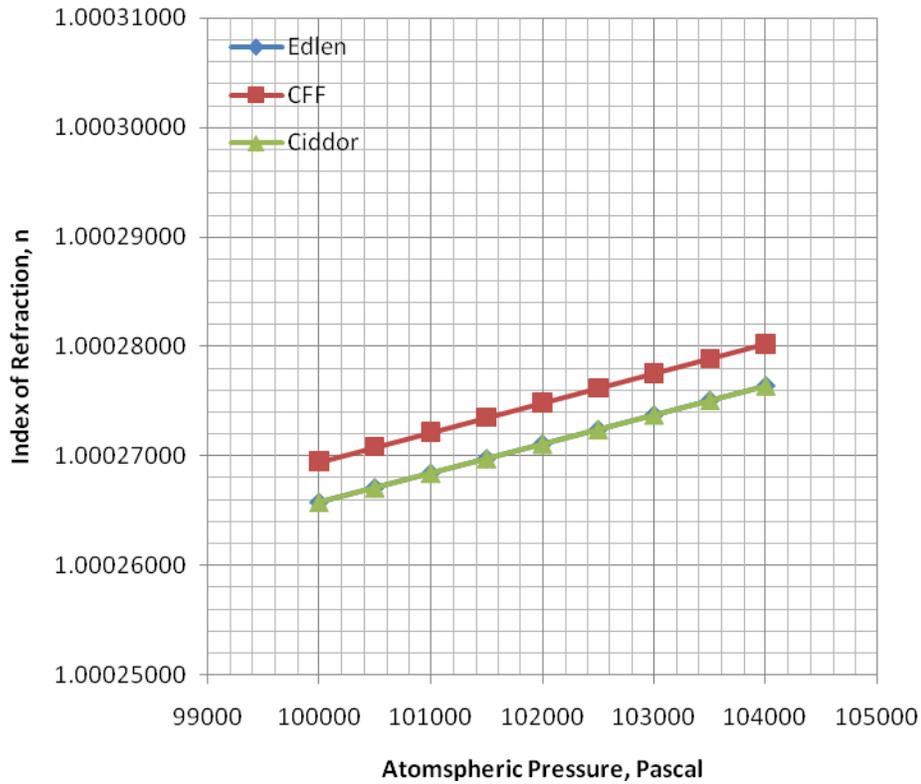


Figure 2. 5 Change of refraction index as a function of atomspheric pressure

Table 2.3 Data used in Figure 2.5

( $\lambda=0.915 \mu\text{m}$ ,  $T=20 \text{ C}^\circ$ ,  $h=56$  and  $\text{CO}_2=380 \text{ ppm}$ )

Pressure (Pa)	Edlen	CFF	Ciddor
100000	1.000265765	1.000269442	1.00026575
100500	1.000267097	1.000270792	1.000267082
101000	1.000268429	1.000272142	1.000268414
101500	1.00026976	1.000273491	1.000269745
102000	1.000271092	1.000274841	1.000271077
102500	1.000272424	1.000276191	1.000272409
103000	1.000273756	1.00027754	1.00027374
103500	1.000275088	1.00027889	1.000275072
104000	1.000276419	1.00028024	1.000276404

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### Example 2.2

Calculate the actual electromagnetic propagating rate and wavelength of a near-infrared laser beam with a wavelength of 1064 nm (in a vacuum) and frequency of  $2.8176 \times 10^8$  MHz through an atmosphere with 30 C° temperature, 56% humidity and 101.3 kPa atmospheric pressure using: a) the CCF; and b) the modified Edlen formulas. If the distance to be measured is 10 km and the maximum error allowed is 5 mm, are the difference in adjustments to atmospheric conditions based on the two methods within the accuracy requirement?

Solutions:

a). Before the calculation, change units to required forms:

$$\lambda = 1064 \text{ nm} = 0.1064 \text{ } \mu\text{m}, P = 101.3 \text{ kPa} = 1013 \text{ hPa}$$

From Eq. 2.9:

$$\begin{aligned} N_g &= 287.6155 + \frac{4.88660}{\lambda^2} + \frac{0.06800}{\lambda^4} \\ &= 287.6155 + \frac{4.88660}{(1.064)^2} + \frac{0.06800}{(1.064)^4} \\ &= 291.985 \end{aligned}$$

From Eq. 2.8

$$\begin{aligned} P_v &= 10^{\left[ \frac{7.5t}{237.3+t} + 0.7858 - 2 \right]} h \\ &= 10^{\left[ \frac{7.5 \times 30}{237.3+30} + 0.7858 - 2 \right]} \times 56 \\ &= 23.74318 \text{ hPa} \end{aligned}$$

The index of refraction is obtained from Eq. 2.10:

$$\begin{aligned}
 n &= 1 + 10^{-6} \left( \frac{273.15 N_g P}{1013.25(t + 273.15)} - \frac{11.27 P_v}{t + 273.15} \right) \\
 &= 1 + 10^{-6} \left( \frac{273.15 \times 291.981 \times 1013}{1013.25(30 + 273.15)} - \frac{11.27 \times 23.74318}{30 + 273.15} \right) \\
 &= 1.000262142
 \end{aligned}$$

The actual electromagnetic propagating rate is:

$$V = \frac{c}{n} = \frac{299792458}{1.000262142} = 2997138904 \text{ m/sec}$$

The actual wavelength is:

$$\lambda = \frac{V}{f} = \frac{2997138904}{2.8176 \times 10^{14}} = 1063.720508 \text{ nm}$$

b). Calculate the index of refraction with the modified Elden formulas from Eq. 2.4 through 2.7:

$$S = \frac{1}{\lambda^2} = \frac{1}{1.064^2} = 0.883317316$$

$$\begin{aligned}
 n_s &= 1 + 10^{-8} \left( 8342.54 + \frac{2406147}{130 - S} + \frac{15998}{38.9 - S} \right) \\
 &= 1 + 10^{-8} \left( 8342.54 + \frac{2406147}{130 - 0.883317316} + \frac{15998}{38.9 - 0.883317316} \right) \\
 &= 1.000273988
 \end{aligned}$$

$$\begin{aligned}
X &= \frac{1+10^{-8}(0.601-0.00972t)P}{1+0.00366t} \\
&= \frac{1+10^{-8}(0.601-0.00972 \times 30) \times 101300}{1+0.003661 \times 30} \\
&= 0.901321303
\end{aligned}$$

$$\begin{aligned}
n &= 1 + \frac{P(n_s - 1)X}{96095.43} - 2.9275 \times 10^{-8} \frac{(3.7345 - 0.04015)}{t + 273.15} P_v \\
&= 1 + \frac{101300 \times (1.00027398 - 1) \times 0.901321303}{96095.43} \\
&\quad - 2.9275 \times 10^{-8} \frac{(3.7345 - 0.04015)}{30 + 273.15} \times 2374.318 \\
&= 1.00025948
\end{aligned}$$

The actual electromagnetic propagating rate is:

$$V = \frac{c}{n} = \frac{299792458}{1.00025948} = 299714688 \text{ m/sec}$$

The actual wavelength is:

$$\lambda = \frac{V}{f} = \frac{299714688}{2.8176 \times 10^{14}} = 1063.723339 \text{ nm}$$

To find whether the difference between the adjustments from the two methods is within the accuracy limit, let's use the Edlen method as a reference to find the time first:

$$t = \frac{10000}{299714688} = 3.3365064 \times 10^{-5} \text{ sec}$$

Multiply the time by the velocity obtained from the CFF to obtain the distance:

$$D = 2997138904 \times 3.3365064 \times 10^{-5} = 9999.97339 \text{ m}$$

The difference between the methods is:

$$10000 - 9999.97339 = 0.02661 \text{ m} = 26.61 \text{ mm}$$

which far exceeds 5 mm, the maximum error allowed. Since the modified Edlen method is considered a more accurate method, the CFF is not appropriate in this case.

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### 2.3 *Laser Fundamentals*

Laser is the acronym for *Light Amplification by Stimulated Emission of Radiation*. Laser light obeys the same rules and laws as any other type of lights. However, unlike natural light such as sun light which is composed of waves with different wavelengths, moving in different directions and in different phases (Figure 2.6), laser light is made of waves with same wavelength, moving in the same direction and in phase (Figure 2.7). These properties of laser light are referred as monochromaticity, collimation and coherence.

To understand the fundamentals about laser light, a brief discussion of how light is produced is necessary. According to quantum mechanics, photons (light) are emitted when atoms with higher energy levels drop to lower energy levels. The difference between the upper and lower energy level determines the wavelength. For most atom types, many upper and lower energy levels exist and as a result, they are capable of emitting photons with many wavelengths. Under room temperature, however, the number of atoms for many types of materials at the upper energy level is very low (the number for a mole of mercury, for example, is  $10^{-57}$ !) and hence almost no light is emitted. When temperature increases, the population of atoms with higher energy levels will increase, resulting more photons being emitted (hence the glow of steel when heated to 2000 K).

Figure 2.8 is a sketch showing the components of a laser system. It consists of a lasing medium, pumping system, reflector and output coupler. The lasing medium is responsible for emitting photons in the laser beam, the pumping system provides energy for the lasing medium and makes lasing possible by creating a population inversion, a condition in which the number of atoms in upper energy levels exceeds the number in lower energy levels. The lasing materials used by various laser devices include gases such as HeNe, solid state materials such as ruby and

YAG and semiconductors such as Gallium Arsenide. Most of the 3D laser scanners employ Nd:YAG - neodymium-doped yttrium aluminum garnet,  $\text{Nd:Y}_3\text{Al}_5\text{O}_{12}$ . The pumping sources for this type of lasing medium can be flash lamp, arc lamp or semiconductor diode laser, with the first two mostly used for large YAD lasers. Semiconductor diode laser is the most efficient pumping sources for low power YAD lasers and is the pumping source for most 3D laser scanners. The reflector is a mirror that is fully reflecting while the output coupler is partially reflecting. For lower power laser such as those used in 3D laser scanner, the lasing medium, reflector and output coupler are usually integrated into one piece.

The lasing process works like this: the pumping source creates population inversion, some of the atoms in the upper energy level drop to the lower level and in the process emit photons, and a photon of correct wavelength will stimulate the excited atom to emit a photon of exactly the wavelength and phase, creating the coherent and monochromatic properties. One stimulates two, two stimulate four and so on, and thus light of a specific wavelength in the lasing medium is amplified by this stimulated emission process and produces the laser beam.

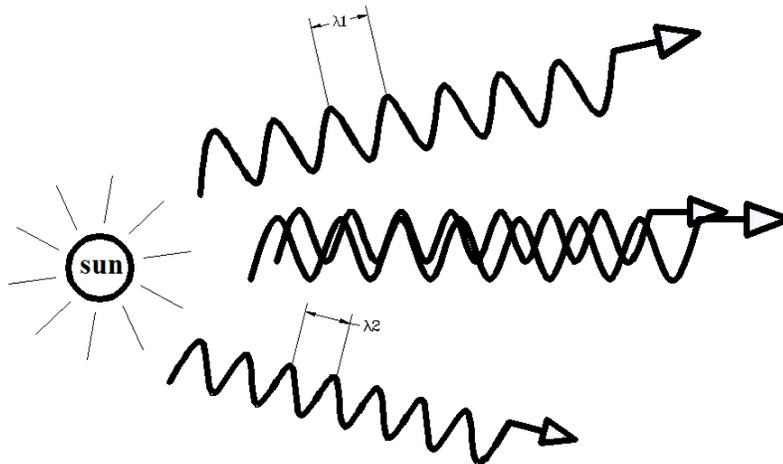


Figure 2.6 Propagation of natural light.

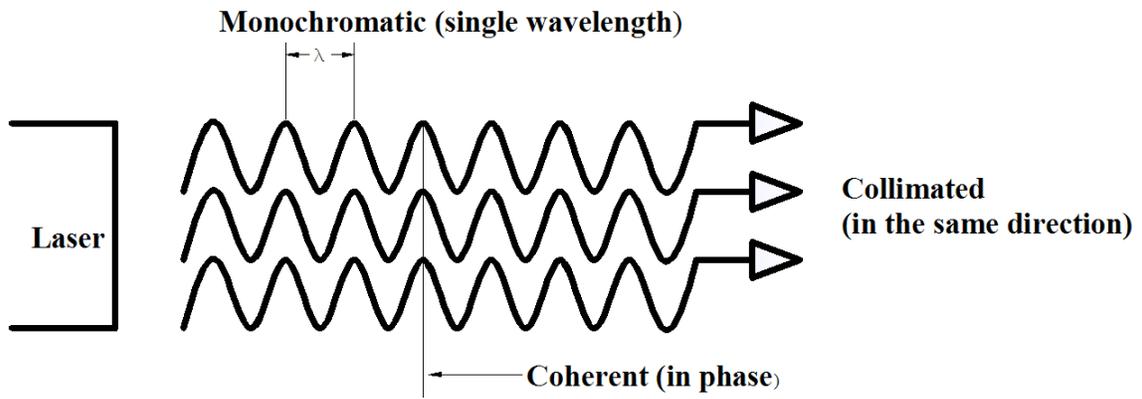


Figure 2.7 Propagation of laser light.

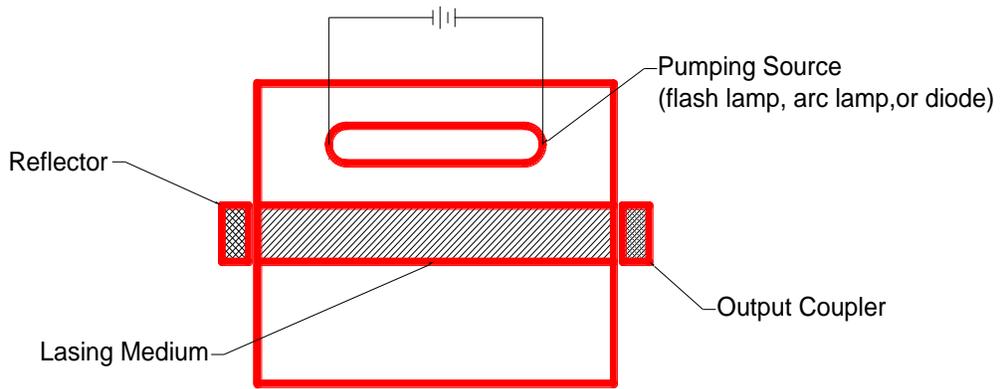


Figure 2.8 Structure of a lasing system.

## **2.4 Electronic Distance Measurement Methods**

In general, two methods are used to determine the distance in electronic distance measurement: Phase-Shift and Time-of-Flight (TOF). Most of the EDM systems used in traditional total stations are based on the *phase-shift* distance determination technique. In this method, a light beam emitted by a diode is split into an external beam which bounces back from the target (prism) to be measured and an internal reference beam and, the phase difference between the two is determined. In addition to the phase difference, the number of full cycles that a light wave has undergone must also be known before the full distance can be calculated. The determination of the number of full cycles is referred to as resolving the cycle ambiguity. The ambiguity is usually solved by using different modulation frequencies. Most of these instruments require the use of a reflector (prism).

In recent years, with the improvement of signal processing technology and precision, the *Time-of-Flight (TOF)* total stations also begin to appear. In these instruments, the EDM system generates many short infrared or laser light pulses, which are transmitted through the telescope to a target. These pulses bounce back from the target and return to the instrument, where the time for the round-trip is determined directly by the system for each light pulse. With the velocity of the light through a medium known, the distance between the instrument and target can be easily determined. The pulses generated by the TOF instruments can be many times more powerful than the energy used for a phase-shift instrument, and hence the TOF method can achieve a much longer distance measurement. Taking the Trimble 5600 DR series of total stations as an example, the Trimble 5600 DR 300+ with the TOF technology can measure up to 400 meters, reflecting off a concrete surface while the Trimble DR Standard employing the phase-shift method can only measure up to 50 meters off the same surface. When a reflector (prism) is used, however, the ranges for the two instruments are comparable, 5500 m for the DR 300+ and 5000 m for the DR Standard. Because of its long range, the TOF method is preferred over phase-shift for reflectorless measurement. The phase-shift technology did have one advantage over TOF: it can achieve better accuracy. However, with the improvement of signal processing technology in TOF method, the accuracy discrepancy between the two is becoming insignificant in many applications.

Currently, most long-range 3D laser scanners employ the TOF method.

## **2.5 Terrestrial 3D Laser Scanning System**

A 3D laser scanner generally has four major components as showing Figure 2.9: a laser unit, a deflecting/rotating unit, a ranging unit and a control and data recording unit. The laser unit produces the laser beam or pulse that is needed for measurement.

Since the laser beam or pulse produced by the laser unit is in only one direction, the scanner needs a mechanism to scan in different directions so that it can cover a large area for each scan. This is achieved by either a rotating polygon mirror to deflect the laser beam to different directions or rotating the entire scanner body. In general, the vertical field of view is usually achieved by rotating deflection mirror while the horizontal one can be obtained by either method.

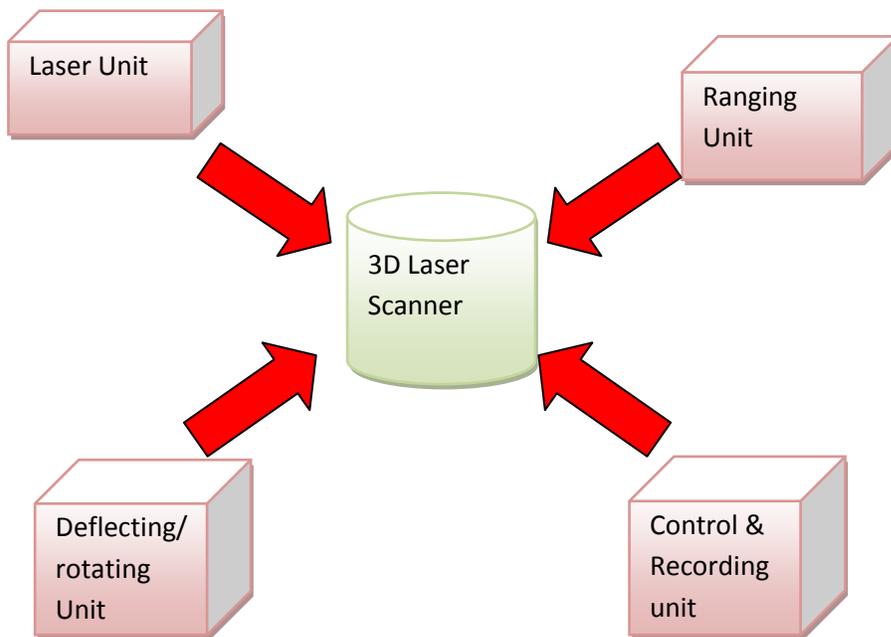


Figure 2.9 Components of a 3D Laser Scanning System

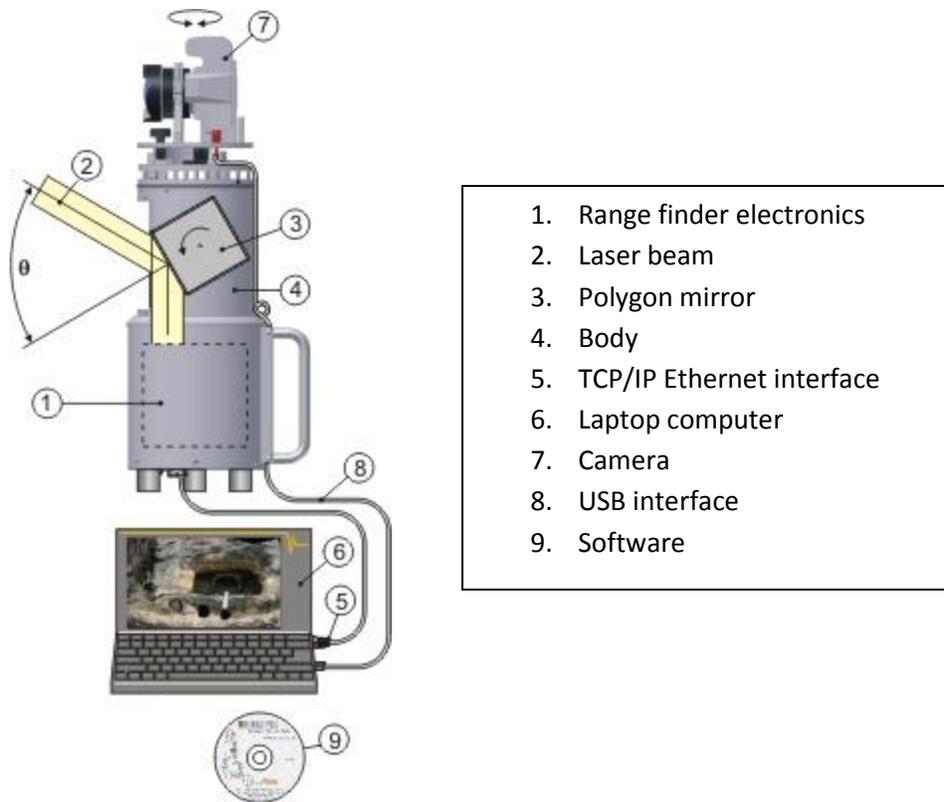
The ranging unit consists of the circuitry for signal detecting and processing to determine the distances and angles. The control and data recording unit is for operation control and data recording. In general, this unit is separate from the scanner body in the form of a data collector or notebook computer.

In addition to hardware, software is also an integral part of 3D laser scanning and can be divided into field and office software. Field software is generally installed on the control and data recording unit while office software is used on an office computer for post-processing.

The maximum distance that a 3D laser scanner can measure depends on the reflectance of the surface, ranging from 100 m to 800 m. For example, the Trimble GX 3D scanner has a range of 350 m if the reflectance of the surface is 90% and 155 m if it is 18%.

Accuracies of 3D laser scanners depend on range, among other factors. At the measurement distance of 100 m, a single point accuracy of several millimeters can be achieved.

Figure 2.10 shows diagram for the RIEGL Z-Series of terrestrial 3D laser scanners.



1. Range finder electronics
2. Laser beam
3. Polygon mirror
4. Body
5. TCP/IP Ethernet interface
6. Laptop computer
7. Camera
8. USB interface
9. Software

Figure 2.10 A diagram for the RIEGL Z-Series of terrestrial 3D laser scanners (Courtesy of RIEGL Laser Measurement System).

## Questions:

1. Explain the differences between phase-shift and TOF methods used in EDM.
2. What is the index of refraction? What is its impact on EDM?
3. Discuss the impact of temperature, humidity and atmospheric pressure on the index of refraction.
4. What is laser? How is it produced? Discuss the properties of laser.
5. List the major components of a 3D laser scanning system.
6. The green laser used in Trimble GX 3D laser scanner is classified Class 2 Laser Product based on US FDA 21 CFR §1040.10 and has a wavelength of 532 nm. Class 2 lasers emit visible (400 to 700nm) output below 1mW. Estimate the number of photons passing through a point per sec.
7. Review and check Example 2.2.

## APPENDIX OF MODULE 1

### Ciddor Calculation of Index of Refraction

(<http://emtoolbox.nist.gov/Wavelength/Documentation.asp>)

(a) Preliminaries:

- Convert all temperatures to Celsius.
- Convert all pressures to Pascal.
- Calculate the mole fraction  $x_i$  as described previously.

(b) Define constants:

$$w_0 = 295.235 \quad w_1 = 2.6422 \quad w_2 = -0.03238 \quad w_3 = 0.004028$$

[Descriptive Link](#)

(A21)

$$k_0 = 238.0185 \quad k_1 = 5792105 \quad k_2 = 57.362 \quad k_3 = 167917$$

[Descriptive Link](#)

(A22)

$$a_0 = 1.58123 \times 10^{-6} \quad a_1 = -2.9331 \times 10^{-8} \quad a_2 = 1.1043 \times 10^{-10}$$

[Descriptive Link](#)

(A23)

$$b_0 = 5.707 \times 10^{-6} \quad b_1 = -2.051 \times 10^{-8}$$

[Descriptive Link](#)

(A24)

$$c_0 = 1.9898 \times 10^{-4} \quad c_1 = -2.376 \times 10^{-6}$$

[Descriptive Link](#)

(A25)

$$d = 1.83 \times 10^{-11} \quad e = -0.765 \times 10^{-8}$$

[Descriptive Link](#)

(A26)

$$P_{R1} = 101325 \quad T_{R1} = 288.15$$

[Descriptive Link](#)

(A27)

$$Z_a = 0.9995922115$$

[Descriptive Link](#)

(A28)

$$P_{v3} = 0.00985938$$

[Descriptive Link](#)

(A29)

$$R = 8.314472 \quad M_v = 0.018015$$

[Descriptive Link](#)

(A30)

(c) Convert the laser vacuum wavelength  $\lambda$  to micrometers and then find

$$S = 1 / \lambda^2$$

[Descriptive Link](#)

(d) Calculate intermediate results that depend on S:

$$r_{a3} = 10^{-8} \{ [k_1 / (k_0 - S)] + [k_3 / (k_2 - S)] \}$$

[Descriptive Link](#)

(A32)

$$r_{v3} = 1.022 \times 10^{-8} [w_0 + w_1 S + w_2 S^2 + w_3 S^3]$$

[Descriptive Link](#)

(A33)

(e) Given the CO<sub>2</sub> concentration  $x_{CO_2}$  in  $\mu\text{mol/mol}$ , calculate

$$M_a = 0.0289635 + 1.2011 \times 10^{-8} (x_{CO_2} - 400)$$

(A34)

[Descriptive Link](#)

$$r_{axs} = r_{as} \left[ 1 + 5.34 \times 10^{-7} (x_{CO_2} - 450) \right]$$

(A35)

[Descriptive Link](#)

(f) Find the Kelvin temperature, the compressibility, and density components:

$$T = t + 273.15$$

(A36)

[Descriptive Link](#)

$$Z_m = 1 - (p/T) \left[ a_0 + a_1 t + a_2 t^2 + (b_0 + b_1 t) x_v + (c_0 + c_1 t) x_v^2 \right] + (p/T)^2 (d + e x_v^2)$$

(A37)

[Descriptive Link](#)

$$\rho_{axs} = p_{xl} M_a / (Z_a R T_{xl})$$

(A38)

[Descriptive Link](#)

$$\rho_v = x_v p M_v / (Z_m R T)$$

(A39)

[Descriptive Link](#)

$$\rho_a = (1 - x_v) p M_a / (Z_m R T)$$

(A40)

[Descriptive Link](#)

(g) Calculate the index of refraction  $n$ :

$$n = 1 + (\rho_a / \rho_{axs}) r_{axs} + (\rho_v / \rho_{vs}) r_{vs}$$

[Descriptive Link](#)

